

HARMONIC-SUPERSPACE TRANSFORM FOR $N=3$ SYM-EQUATIONS

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Abstract

The $SU(3)/U(1) \times U(1)$ harmonic variables are used in the harmonic-superspace representation of the $D=4$, $N=3$ SYM-equations. The harmonic superfield equations of motion in the simple non-covariant gauge contain the nilpotent harmonic analytic connections. It is shown that these harmonic SYM-equations are equivalent to the finite set of solvable linear iterative equations.

1 Introduction

The $SU(2)/U(1)$ harmonic superspace (HSS) has been used to solve the $D = 4$, $N = 2$ off-shell constraints [1, 2]. The integrability interpretation of the dual version of the equations of motion has been also formulated for the $N = 2$ supersymmetric Yang-Mills (SYM) theory in the framework of this superspace [3].

In the HSS-approach to the $D = 4$, $N = 3$ supersymmetry, the $SU(3)/U(1) \times U(1)$ harmonics have been used for the off-shell description of the corresponding SYM-theory [4]. The $N = 3$ SYM-equations in the ordinary superspace [5] have been transformed to the zero-curvature equations for the harmonic gauge connections, however, nobody tried earlier to solve these equations. Moreover, it has been shown that some set of analytic connections does not generate any solutions in the ordinary superspace [6].

We propose to solve first the harmonic zero-curvature equations in terms of the independent non-analytic superfield matrix v (bridge) describing the transform between different representations of the gauge group. Then we analyze the dynamical Grassmann (G-) analyticity equations for the composed on-shell harmonic connections constructed in terms of this superfield v .

It will be shown that the special non-covariant gauge choice for the matrix v simplifies drastically the solution of all equations. The crucial feature of our gauge is the nilpotency of v and the corresponding G-analytic harmonic connections. The Lorenz invariance of the SYM-equations is broken down to the $SO(1,1)$ subgroup in this gauge. We demonstrate that the 1-st order harmonic bridge equations with the nilpotent HSS-connections produce also the linear 2-nd order differential constraints for the bridge matrix.

Thus, we show that the $N = 3$ SYM-equations in the ordinary superspace can be transformed to the linear solvable matrix differential equations in the harmonic superspace. This method allow us to analyze also solutions of the dimensionally reduced SYM-equations with 12 supercharges.

2 Harmonic representation of $D = 4$, $N = 3$ SYM constraints

We shall consider the non-covariant $SO(1, 1) \times SU(3)$ representation of the $D = 4$, $N = 3$ superspace coordinates z

$$\begin{aligned} x^{1\dot{1}} \equiv x^\pm = t + x^3, \quad x^{2\dot{2}} \equiv x^\mp = t - x^3, \quad x^{1\dot{2}} \equiv y = x^1 + ix^2, \quad x^{2\dot{1}} \equiv \bar{y} = x^1 - ix^2, \\ \theta_i^1 = \theta_i^+, \quad \theta_i^2 = \theta_i^-, \quad \bar{\theta}^{i\dot{1}} = \bar{\theta}^{i+}, \quad \bar{\theta}^{i\dot{2}} = \bar{\theta}^{i-}. \end{aligned} \quad (2.1)$$

where $1, 2, \dot{1}, \dot{2}$ are the $SL(2, C)$ indices, $i = 1, 2$ and 3 are indices of the fundamental representations of the group $SU(3)$. The $SO(1, 1)$ weights of these coordinates are $0, \pm 1, \pm 2$, respectively.

Let us introduce the $(4|6, 6)$ -dimensional superspace gauge connections $A(z)$ and the corresponding covariant derivatives ∇

$$\begin{aligned} \nabla_\pm^i = D_\pm^i + A_\pm^i, \quad \bar{\nabla}_{i\pm} = \bar{D}_{i\pm} + \bar{A}_{i\pm}, \\ \nabla_\mp = \partial_\mp + A_\mp, \quad \nabla_- = \partial_- + A_-, \quad \nabla_y = \partial_y + A_y, \quad \bar{\nabla}_y = \bar{\partial}_y + \bar{A}_y, \end{aligned} \quad (2.2)$$

where the space-time derivatives and the Grassmann derivatives are considered.

The $D = 4$, $N = 3$ SYM-constraints [5] have the following reduced-symmetry form:

$$\{\nabla_+^k, \nabla_+^l\} = 0, \quad \{\bar{\nabla}_{k+}, \bar{\nabla}_{l+}\} = 0, \quad \{\nabla_+^k, \bar{\nabla}_{l+}\} = 2i\delta_l^k \nabla_\mp, \quad (2.3)$$

$$\{\nabla_+^k, \nabla_-^l\} = \bar{W}^{kl}, \quad \{\nabla_+^k, \bar{\nabla}_{l-}\} = 2i\delta_l^k \nabla_y, \quad (2.4)$$

$$\{\nabla_-^k, \bar{\nabla}_{l+}\} = 2i\delta_l^k \bar{\nabla}_y, \quad \{\bar{\nabla}_{k+}, \bar{\nabla}_{l-}\} = W_{kl}, \quad (2.5)$$

$$\{\nabla_-^k, \nabla_-^l\} = 0, \quad \{\bar{\nabla}_{k-}, \bar{\nabla}_{l-}\} = 0, \quad \{\nabla_-^k, \bar{\nabla}_{l-}\} = 2i\delta_l^k \nabla_-, \quad (2.6)$$

where W^{kl} and \bar{W}^{kl} are the gauge-covariant superfields constructed from the gauge connections. The equations of motion for the superfield strengthes follow from the Bianchi identities.

Let us analyze first Eqs.(2.3) which can be treated as integrability conditions for the positive-helicity connections. These conditions have the following pure gauge solution:

$$\nabla_+^k = g^{-1} D_+^k g, \quad \bar{\nabla}_{k+} = g^{-1} \bar{D}_{k+} g, \quad \nabla_\mp = g^{-1} \partial_\mp g. \quad (2.7)$$

Using the on-shell gauge condition $g = 1$ we can obtain the simple general solution of Eqs.(2.3)

$$A_+^k = 0, \quad \bar{A}_{k+} = 0, \quad A_\mp = 0. \quad (2.8)$$

The analogous gauge conditions excluding the part of connections has been considered in Ref.[7] for the self-dual 4D SYM-theory and in Ref.[8] for the 10D SYM equations.

The $SU(3)/U(1) \times U(1)$ harmonics [4] parameterize the corresponding coset space. They form an $SU(3)$ matrix u_i^I and are defined modulo $U(1) \times U(1)$ transformations

$$u_i^1 = u_i^{(1,0)}, \quad u_i^2 = u_i^{(-1,1)}, \quad u_i^3 = u_i^{(0,-1)}, \quad (2.9)$$

where i is the index of the triplet representation of $SU(3)$. The complex conjugated harmonics have opposite $U(1)$ charges

$$u_1^i = u^{i(-1,0)} , \quad u_2^i = u^{i(1,-1)} , \quad u_3^i = u^{i(0,1)} . \quad (2.10)$$

These harmonics satisfy the following relations:

$$\begin{aligned} u_i^I u_J^i &= \delta_J^I , \quad u_i^I u_I^k = \delta_i^k , \\ \varepsilon^{ikl} u_i^1 u_k^2 u_l^3 &= 1 . \end{aligned} \quad (2.11)$$

The $SU(3)$ -invariant harmonic derivatives act on the harmonics

$$\begin{aligned} \partial_J^I u_i^K &= \delta_J^K u_i^I , \quad \partial_J^I u_K^i = -\delta_K^I u_J^i , \\ [\partial_J^I, \partial_L^K] &= \delta_J^K \partial_L^I - \delta_L^K \partial_J^I . \end{aligned} \quad (2.12)$$

We shall use the special $SU(3)$ -covariant conjugation

$$\widetilde{u_i^1} = u_i^3 , \quad \widetilde{u_i^3} = u_i^1 , \quad \widetilde{u_i^2} = -u_i^2 . \quad (2.13)$$

The corresponding conjugation of the harmonic derivatives is

$$\widetilde{\partial_3^1 f} = -\partial_3^1 \widetilde{f} , \quad \widetilde{\partial_2^1 f} = \partial_3^2 \widetilde{f} , \quad (2.14)$$

where $f(u)$ is a harmonic function.

We can define the real analytic harmonic superspace $H(4, 6|4, 4)$ with 6 coset harmonic dimensions and the following left and right coordinates:

$$\begin{aligned} \zeta &= (\xi^\pm , \xi^\mp , \xi , \bar{\xi} , \theta_2^\pm , \theta_3^\pm , \bar{\theta}^{1\pm} , \bar{\theta}^{2\pm}) , \\ \xi^\pm &= x^\pm + i(\theta_3^\pm \bar{\theta}^{3\mp} - \theta_1^\pm \bar{\theta}^{1\mp}) , \quad \xi^\mp = x^\mp + i(\theta_3^\mp \bar{\theta}^{3\mp} - \theta_1^\mp \bar{\theta}^{1\mp}) , \\ \xi &= y + i(\theta_3^+ \bar{\theta}^{3-} - \theta_1^+ \bar{\theta}^{1-}) , \quad \bar{\xi} = \bar{y} + i(\theta_3^- \bar{\theta}^{3+} - \theta_1^- \bar{\theta}^{1+}) , \end{aligned} \quad (2.15)$$

where $\theta_I^\pm = \theta_k^\pm u_I^k$, $\bar{\theta}^{\pm I} = \bar{\theta}^{\pm k} u_k^I$.

The CR-structure in $H(4, 6|4, 4)$ involves the G-derivatives

$$D_\pm^1 , \bar{D}_{3\pm} , \quad (2.16)$$

which commute with the harmonic derivatives D_2^1 , D_3^2 and D_3^1 .

These derivatives have the following explicit form in the analytic coordinates:

$$D_\alpha^{(1,0)} \equiv D_\alpha^1 = \partial_\alpha^1 \equiv \partial/\partial\theta_1^\alpha \quad \bar{D}_\alpha^{(0,1)} \equiv \bar{D}_{3\alpha} = \partial_{3\alpha} \equiv \partial/\partial\bar{\theta}^{3\alpha} , \quad (2.17)$$

$$\begin{aligned} D^{(2,-1)} &\equiv D_2^1 = \partial_2^1 + \frac{i}{2}\theta_2^+ \bar{\theta}^{1+} \partial_+ + \frac{i}{2}\theta_2^+ \bar{\theta}^{1-} \partial_\xi + \frac{i}{2}\theta_2^- \bar{\theta}^{1+} \bar{\partial}_\xi + \frac{i}{2}\theta_2^- \bar{\theta}^{1-} \partial_- \\ &\quad - \theta_2^+ \partial_+^1 - \theta_2^- \partial_-^1 + \bar{\theta}^{1+} \bar{\partial}_{2+} + \bar{\theta}^{1-} \bar{\partial}_{2-} , \\ D^{(-1,2)} &\equiv D_3^2 = \partial_3^2 + \frac{i}{2}\theta_3^+ \bar{\theta}^{2+} \partial_+ + \frac{i}{2}\theta_3^+ \bar{\theta}^{2-} \partial_\xi + \frac{i}{2}\theta_3^- \bar{\theta}^{2+} \bar{\partial}_\xi + \frac{i}{2}\theta_3^- \bar{\theta}^{2-} \partial_- \\ &\quad - \theta_3^+ \partial_+^2 - \theta_3^- \partial_-^2 + \bar{\theta}^{2+} \bar{\partial}_{3+} + \bar{\theta}^{2-} \bar{\partial}_{3-} , \end{aligned} \quad (2.18)$$

$$\begin{aligned} D^{(1,1)} &\equiv D_3^1 = \partial_3^1 + i\theta_3^+ \bar{\theta}^{1+} \partial_+ + i\theta_3^+ \bar{\theta}^{1-} \partial_\xi + i\theta_3^- \bar{\theta}^{1+} \bar{\partial}_\xi + i\theta_3^- \bar{\theta}^{1-} \partial_- \\ &\quad - \theta_3^+ \partial_+^1 - \theta_3^- \partial_-^1 + \bar{\theta}^{1+} \bar{\partial}_{3+} + \bar{\theta}^{1-} \bar{\partial}_{3-} , \end{aligned} \quad (2.19)$$

where $\partial_{\pm} = \partial/\partial\xi^{\pm}$, $\partial_{\pm} = \partial/\partial\xi^{\pm}$, $\partial_{\xi} = \partial/\partial\xi$ and $\bar{\partial}_{\xi} = \partial/\partial\bar{\xi}$.

It is crucial that we start from the specific gauge conditions (2.8) for the $N = 3$ SYM-connections which break $SL(2, C)$ but preserve the $SU(3)$ -invariance. Consider the harmonic transform of the covariant Grassmann derivatives in this gauge using the projections on the $SU(3)$ -harmonics

$$\nabla_{+}^I \equiv u_i^I D_{+}^i = D_{+}^I, \quad \bar{\nabla}_{I+} \equiv u_I^i \bar{D}_{i+} = \bar{D}_{I+}, \quad \{D_{+}^I, \bar{D}_{K+}\} = 2i\delta_K^I \partial_{\pm}, \quad (2.20)$$

$$\nabla_{-}^I \equiv u_i^I \nabla_{-}^i = D_{-}^I + \mathcal{A}_{-}^I, \quad \bar{\nabla}_{I-} \equiv u_I^i \bar{\nabla}_{i-} = \bar{D}_{I-} + \bar{\mathcal{A}}_{I-}, \quad (2.21)$$

where the harmonized Grassmann connections \mathcal{A}_{-}^I and $\bar{\mathcal{A}}_{I-}$ are defined.

The $SU(3)$ -harmonic projections of the superfield constraints (2.4-2.6) can be derived from the basic set of the $N = 3$ super-integrability conditions for two components of the harmonized connection:

$$D_{+}^1 \mathcal{A}_{-}^1 = \bar{D}_{3+} \mathcal{A}_{-}^1 = D_{+}^1 \bar{\mathcal{A}}_{3-} = \bar{D}_{3+} \bar{\mathcal{A}}_{3-} = 0 \quad (2.22)$$

$$D_{-}^1 \mathcal{A}_{-}^1 + (\mathcal{A}_{-}^1)^2 = 0, \quad \bar{D}_{3-} \bar{\mathcal{A}}_{3-} + (\bar{\mathcal{A}}_{3-})^2 = 0, \quad (2.23)$$

$$D_{-}^1 \bar{\mathcal{A}}_{3-} + \bar{D}_{3-} \mathcal{A}_{-}^1 + \{\mathcal{A}_{-}^1, \bar{\mathcal{A}}_{3-}\} = 0.$$

All projections of the SYM-equations can be obtained by the action of the harmonic $SU(3)$ derivatives D_K^I on these basic conditions.

These Grassmann zero-curvature equations have the very simple general solution

$$\mathcal{A}_{-}^1(v) = e^{-v} D_{-}^1 e^v, \quad \bar{\mathcal{A}}_{3-}(v) = e^{-v} \bar{D}_{3-} e^v, \quad (2.24)$$

where *the bridge* v is the gauge-Lie-algebra valued (5,5)-superfield matrix satisfying the additional constraint

$$(D_{+}^1, \bar{D}_{3+})v = 0. \quad (2.25)$$

Consider the gauge transformations of the bridge

$$e^v \Rightarrow e^{\lambda} e^v e^{\tau_r}, \quad (2.26)$$

where $\lambda \in H(4, 6|4, 4)$ is the (4,4)-analytic matrix parameter, and the parameter τ_r does not depend on harmonics. The matrix e^v realizes the harmonic transform of the gauge superfields $A_{\pm}^k, \bar{A}_{k\pm}$ in the central basis (CB) to the harmonic gauge superfields of the analytic basis (AB) using the analytic λ -representation of the gauge group.

The bridge v defines the on-shell harmonic connections of the analytic basic

$$V_K^I(v) = e^v D_K^I e^{-v} \quad (2.27)$$

which satisfy by construction the harmonic zero-curvature equations, for instance,

$$D_2^1 V_3^2 - D_3^2 V_2^1 + [V_2^1, V_3^2] - V_3^1 = 0, \quad (2.28)$$

$$D_2^1 V_3^1 - D_3^1 V_2^1 + [V_2^1, V_3^1] = 0.$$

The dynamical SYM-equations in the bridge representation (2.24) are reduced to the harmonic differential conditions for the basic Grassmann connections:

$$(D_2^1, D_3^2, D_3^1) (\mathcal{A}_{-}^1(v), \bar{\mathcal{A}}_{3-}(v)) = 0, \quad (2.29)$$

which are equivalent to the following set of the dynamic G-analyticity equations:

$$(D_-^1, \bar{D}_{3-}) \left(V_2^1(v), V_3^2(v), V_3^1(v) \right) = 0 . \quad (2.30)$$

The positive-helicity analyticity conditions

$$(D_+^1, \bar{D}_{3+}) \left(V_2^1(v), V_3^2(v), V_3^1(v) \right) = 0 \quad (2.31)$$

are satisfied automatically for the bridge in the gauge (2.25). Stress that the analyticity equations in the bridge representation describe all SYM-solutions without a loss of generality.

The inverse harmonic transform determines the harmonic-independent on-shell gauge solutions in the ordinary superspace

$$\begin{aligned} \mathcal{A}_-^1(v) &\Rightarrow A_-^i(v) = (u_1^i + u_2^i D_1^2 + u_3^i D_1^3) e^{-v} D_-^1 e^v \\ \bar{\mathcal{A}}_{3-}(v) &\Rightarrow \bar{A}_{i-}(v) = (u_i^3 - u_i^1 D_1^3 - u_i^2 D_2^3) e^{-v} \bar{D}_{3-} e^v . \end{aligned} \quad (2.32)$$

By construction, these superfields satisfy the $D = 4$, $N = 3$ CB-constraints (2.4-2.6) and the harmonic differential conditions

$$D_K^I \left(A_-^i(v), \bar{A}_{i-}(v) \right) = 0 , \quad (2.33)$$

if the relations (2.29) are fulfilled.

It is useful to calculate the Grassmann connection $\mathcal{A}_-^1(v)$ in terms of the basic analytic matrices

$$e^{-v} D_-^1 e^v = b^1 - \theta_1^- (b^1)^2 + \bar{\theta}^{3-} \left(d_3^1 - \frac{1}{2} \{b^1, c_3\} \right) + \theta_1^- \bar{\theta}^{3-} \left([b^1, d_3^1] + \frac{1}{2} [c_3, (b^1)^2] \right) . \quad (2.34)$$

The conjugated connection $\bar{\mathcal{A}}_{3-}(v)$ can be constructed analogously.

3 Harmonic-superspace equations of motion

Using the λ -transformations (2.26) one can gauge away the first terms in the Grassmann decomposition of the bridge v and choose the non-supersymmetric nilpotent gauge for this (5,5)-superfield:

$$v = \theta_1^- b^1 + \bar{\theta}^{3-} c_3 + \theta_1^- \bar{\theta}^{3-} d_3^1 , \quad v^2 = \theta_1^- \bar{\theta}^{3-} [c_3, b^1] , \quad (3.1)$$

$$e^{-v} = I - v + \frac{1}{2} v^2 = I - \theta_1^- b^1 - \bar{\theta}^{3-} c_3 + \theta_1^- \bar{\theta}^{3-} \left(\frac{1}{2} [c_3, b^1] - d_3^1 \right) , \quad (3.2)$$

where the fermionic matrices b^1, c_3 and the bosonic matrix d_3^1 are (4,4)-analytic functions of the coordinates ζ (2.15).

The restrictions on the bridge matrix in the gauge group $SU(n)$

$$\text{Tr } v = 0 , \quad v^\dagger = -v \quad (3.3)$$

are equivalent to the following relations for the analytic matrices b^1, c_3 and d_3^1

$$\text{Tr } b^1 = 0 , \quad \text{Tr } c_3 = 0 , \quad \text{Tr } d_3^1 = 0 , \quad (b^1)^\dagger = c_3 , \quad (d_3^1)^\dagger = -d_3^1 . \quad (3.4)$$

It is useful to construct the on-shell superfield strength in the analytic basis

$$\bar{W}^{12}(b^1) = -D_+^2 b^1 + \theta_2^- D_-^2 D_+^2 b^1 + \theta_2^- [D_1^2 b^1, D_+^2 b^1] + \theta_1^- \theta_2^- [D_-^2 b^1, D_+^2 b^1] , \quad (3.5)$$

which satisfies the non-Abelian conditions of Grassmann and harmonic analyticities.

The dynamical G-analyticity equation (2.30) in the gauge (3.1) is equivalent to the following harmonic differential bridge equation:

$$V_2^1(v) = \theta_2^- b^1 \equiv v_2^1 , \quad (v_2^1)^2 = 0 \quad (3.6)$$

$$D_2^1 e^{-v} = e^{-v} v_2^1 . \quad (3.7)$$

where the analytic nilpotent representation for the on-shell harmonic connection arises.

The 2-nd on-shell harmonic connection is also nilpotent

$$V_3^2(v) = -\bar{\theta}^{2-} c_3 \equiv v_3^2 , \quad (3.8)$$

$$D_3^2 e^{-v} = e^{-v} v_3^2 . \quad (3.9)$$

Underline that the nilpotency of the analytic parts of the harmonic connections V_2^1 and V_3^2 (2.27) follows directly from the gauge condition (2.31), and the analyticity of these connections requires the additional harmonic restrictions on the functions b^1, c_3 and d_3^1 .

The 1-st bridge equation (3.7) generates the following nonlinear equations for the (4,4)-analytic matrices:

$$D_2^1 b^1 = -\theta_2^- (b^1)^2 , \quad (3.10)$$

$$D_2^1 c_3 = -\theta_2^- (d_3^1 + \frac{1}{2} \{b^1, c_3\}) , \quad (3.11)$$

$$D_2^1 d_3^1 = \frac{1}{2} \theta_2^- \left([d_3^1, b^1] + \frac{1}{2} [(b^1)^2, c_3] \right) . \quad (3.12)$$

It is important that all these equations contain the nilpotent element θ_2^- in the non-linear parts, so they can be reduced to the set of linear iterative equations.

The 2-nd bridge equation (3.9) gives us

$$D_3^2 b^1 = \bar{\theta}^{2-} (-d_3^1 + \frac{1}{2} \{b^1, c_3\}) , \quad (3.13)$$

$$D_3^2 c_3 = \bar{\theta}^{2-} (c_3)^2 \quad (3.14)$$

$$D_3^2 d_3^1 = \frac{1}{2} \bar{\theta}^{2-} \left([d_3^1, c_3] + \frac{1}{2} [b^1, (c_3)^2] \right) . \quad (3.15)$$

It is useful to derive the following relations:

$$\theta_2^- D_2^1 (b^1, c_3, d_3^1) = 0 , \quad \bar{\theta}^{2-} D_3^2 (b^1, c_3, d_3^1) = 0 , \quad (3.16)$$

$$\theta_2^- D_3^2 b^1 + \bar{\theta}^{2-} D_2^1 c_3 = \theta_2^- \bar{\theta}^{2-} \{b^1, c_3\} , \quad (3.17)$$

$$\bar{\theta}^{2-} D_2^1 c_3 - \theta_2^- D_3^2 b^1 = 2\theta_2^- \bar{\theta}^{2-} d_3^1 , \quad (3.18)$$

$$\theta_2^- D_3^1 b^1 + \bar{\theta}^{1-} D_2^1 c_3 = -\theta_2^- \theta_3^- (b^1)^2 + \theta_2^- \bar{\theta}^{1-} \{b^1, c_3\} . \quad (3.19)$$

The bridge equations with the nilpotent analytic connections yield the following 2-nd order linear conditions for the coefficient functions

$$D_2^1 D_2^1 (b^1, c_3, d_3^1) = 0 , \quad D_3^2 D_3^2 (b^1, c_3, d_3^1) = 0 . \quad (3.20)$$

Now we consider the iterative procedure of solving the basic non-Abelian harmonic differential equations for the (4,4) analytic matrices b^1 and c_3 using the partial decomposition in the Grassmann variables $\theta_2^-, \theta_3^-, \bar{\theta}^{1-}, \bar{\theta}^{2-}$. The matrix (4,0) coefficients of this decomposition have dimensions $-1/2 \geq l \geq -5/2$

$$\begin{aligned} b^1 = & \beta^1 + \theta_2^- B^{12} + \theta_3^- B^{13} + \bar{\theta}^{1-} B^0 + \bar{\theta}^{2-} B_2^1 + \theta_2^- \theta_3^- \beta^0 + \theta_2^- \bar{\theta}^{1-} \beta^2 + \theta_3^- \bar{\theta}^{2-} \beta_2^{13} \\ & + \theta_3^- \bar{\theta}^{1-} \beta^3 + \bar{\theta}^{1-} \bar{\theta}^{2-} \beta_2 + \theta_2^- \bar{\theta}^{2-} \eta^1 + \theta_2^- \theta_3^- \bar{\theta}^{1-} B^{23} + \theta_3^- \bar{\theta}^{1-} \bar{\theta}^{2-} B_2^3 + \theta_2^- \theta_3^- \bar{\theta}^{2-} C^{13} \\ & + \theta_2^- \bar{\theta}^{1-} \bar{\theta}^{2-} C^0 + \theta_2^- \theta_3^- \bar{\theta}^{1-} \bar{\theta}^{2-} \eta^3 . \end{aligned} \quad (3.21)$$

The first iterative (4,0) equations ($l = -1/2$) are linear and homogeneous

$$\hat{D}_2^1 \beta^1 = \hat{D}_3^2 \beta^1 = 0 . \quad (3.22)$$

The next harmonic iterative equations for the (4,0) components with $l \leq -1/2$ can be resolved on the each step via functions of the highest dimensions or their derivatives. These linear equations contain nonlinear sources constructed from the solutions of the previous equations. Note that some (4,0) iterative equations are pure algebraic relations which reduce the number of independent functions, for instance,

$$B_2^1 = -\hat{D}_2^1 B^0 - \frac{i}{2} \theta_2^+ \partial_\xi \beta^1 , \quad B^{12} = \hat{D}_3^2 B^{13} + \frac{i}{2} \bar{\theta}^{2+} \bar{\partial}_\xi \beta^1 , \quad (3.23)$$

$$\beta^2 = \hat{D}_3^2 \beta^3 - \frac{i}{2} \bar{\theta}^{2+} \bar{\partial}_\xi B^0 , \quad \beta_2^{13} = -\hat{D}_2^1 \beta^3 + \frac{i}{2} \theta_2^+ \partial_\xi B^{13} . \quad (3.24)$$

The finite set of the linear harmonic differential equations for the independent (4,0) functions $\beta^1, B^0, B^{13}, \beta^3 \dots$ can be, in principle, explicitly solved. Thus, the $SU(3)/U(1) \times U(1)$ harmonic method together with the simple gauge conditions allow us to transform the $N = 3$ superfield SYM-constraints to the harmonic differential equations which are equivalent to the finite set of the iterative solvable linear equations.

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